B.Sc. (P) Life-Science / Applied Life- Science
Part-I
Paper No- 107(b)
Mathematics for life-science

Time: 3hs Max Marks: 100
Each question is of equal marks:
Scientific Calculator is allowed.

1 (a) Let l_1 and l_2 be two straight lines in a plane. Then write down all possible values for $l_1 \cap l_2$.

(b) A linear growth function Q(t) = at + b, assume the value Q_1 =88 mg at time instant t_1=14 sec. and the value Q_2=89.5 mg at t_2 =39 sec. Establish the function Q(t) and find the rate of growth.

(c) Discuss the character of the roots in the quadratic equation:

$$9x^2 - (m-3)\,x + 1 = 0$$

(d) Sketch the graphs and discuss their increasing and decreasing behaviour

$y = \text{Cos}2x, \quad 0 \le x \le 2\pi.$

OR

(a) By proper definition men may be assigned to one of the following categories:

A: tall and slim; B: tall and fat
C: short and slim; D: short and fat

These four categories constitute a set χ = { A, B,C,D} . Form the subsets tall, short, fat, slim and find the corresponding complimentary sets.

(b) If a crystal grows in such a way that all linear dimensions increase by 15%. Find the percent increase of its surface and volume.

(c) The rate of growth R of a biological population is given in terms of its size n by: R= k.n (a - n)
where a, k are positive constants . Find the range of R if n < a.

(d) Draw the graph of $y = 1-2x- x^2$. Also find the minimum value of y.

2 (a) An explorer bee discovers a source of honey at noon. This source is located 800m east and 1385.6m (= $800\sqrt{3}$) south from hive. What polar coordinates will the bee signal?

(b) Graph the function $y = \frac{1}{2}e^{-x} - 3$ and its domain and range.

(c) On his 13th birthday a child was 112 cm tall, on his 14th birthday (112 $\times$ 2048) cm tall. Assume a geometric monthly growth rate, what is this rate?

(d) Is 0.9+ 0.09+0.009+0.0009+------------------ = 1?

OR

(a) Graph the function $y = \sin(x + \frac{\pi}{3})$ $\quad -\pi \le x \le \pi$.

(b) Under ideal conditions a certain bacteria population is known to double every three hours. Suppose there are initially 100 bacteria. Estimate the size of the population after 20 hrs.

(c) Define divine ratio.

(d) Is $\frac{2}{3}+\frac{2}{3^2}+\frac{2}{3^3}+-------=1?$

3 (a) (i) $\lim_{x\to\infty}\frac{x^3-2x+3}{5-2x^2}$ (ii) $\lim_{x\to 3}\frac{x^2-x-6}{x-3}$

(b) If $f(x)=\begin{cases}2x-1 & x\le 1\\ -x+2 & x>1\end{cases}$

Is $f(x)$ continuous at x = 1?

(c) Differentiate the following functions w.r.t. x :-

(i) (tan x + sec x) (cot x + cosec x)

(ii) $(x \sin ax)^3$

(d) The size of a slowly growing bacterial culture is approximately given by: $N(t) = N_0 + 10t + 2t^3$, where N_0 is the size when t = 0 and time t is measured in hours. Find the average rate of growth during the time interval t = 1 to t = 5.

OR

(a) (i) $\lim_{x\to 0}\frac{5^x+3^x-2}{x}$ (ii) $\lim_{x\to 3}\frac{x^2-9}{x-3}$

(b) Find constants a and b so that $f(x)$ is continuous for all x,

$$f(x)=\begin{cases}ax+3, & x<1\\ 2, & x=1\\ x^2+b & x>1\end{cases}$$

(c) Differentiate the following functions w.r.t. x :-

(i) $a^x \sin x \log x$ (ii) 3^{x+2}

(d) The size of a slowly growing bacteria culture is approximately given by $N(t) = N_0 + 52t + 2t^2$, where N_0 is the size when t = 0 and time t is measured in hours. Find the growth rate when time is 5 hours.

4 (a) Verify that $\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 z}{\partial y\partial x}$, for the following:

(i) $z = x^4 + y^2 - 2x^2y$

(ii) $z = ax^2 + 2hxy + by^2$

(b) If $u = \log(x^2+y^2)$ find first and second order partial derivatives of u and hence prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = 0$.

(c) Verify that y=at + e^{kt} is a solution of $\frac{\partial y}{\partial x} = ky + a(1 - kt)$.

(d) Show that y= x^3+4x+5 is a solution of the differential equation

$$\frac{\partial^2 y}{\partial x^2} - 6x = 0$$

OR

(a) Verify that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$, for the following:

(i) z = x^2sin (x + y)

(ii) z= y log x

(b) $For\ z = e^{x-y}\ show\ that\ \frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial y \partial x} + \frac{\partial^2 z}{\partial^2 y} = 0$.

(c) Verify that y = sin ? x is a solution of the differential equation:

$$\frac{\partial^2 y}{\partial x^2} + \omega^2 y = 0$$

(d) Show that $y = \frac{c}{x} + d$ is a solution of the differential equation $\frac{\partial y}{\partial x} + \frac{c}{x^2} = 0$